Section 12.8

The Counting Principle and Permutations

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Counting Principle

- If a first experiment can be performed in M distinct ways and a second experiment can be performed in N distinct ways, then the two experiments in that specific order can be performed in $M \cdot N$ distinct ways.

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Example

- A password to logon to a computer system is to consist of 3 letters followed by 3 digits. Determine how many different passwords are possible if:

  a) repetition of letters and digits is permitted

  b) repetition of letters and digits is not permitted

  c) the first letter must be a vowel (a, e, I, o, u), the first digit cannot be 0, and repetition of letters and digits is not permitted.

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Solutions

a. repetition of letters and digits is permitted.

- There are 26 letters and 10 digits. We have 6 positions to fill.

$$\underset{\text{L}}{26}\quad\underset{\text{L}}{26}\quad\underset{\text{L}}{26}\quad\underset{\text{D}}{10}\quad\underset{\text{D}}{10}\quad\underset{\text{D}}{10}$$

$$= 26 \cdot 26 \cdot 26 \cdot 10 \cdot 10 \cdot 10$$

$$= 17{,}576{,}000$$

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Solution

b. repetition of letters and digits is not permitted.

$$\underset{\text{L}}{26}\quad \underset{\text{L}}{25}\quad \underset{\text{L}}{24}\quad \underset{\text{D}}{10}\quad \underset{\text{D}}{9}\quad \underset{\text{D}}{8}$$

$$= 26 \cdot 25 \cdot 24 \cdot 10 \cdot 9 \cdot 8$$
$$= 11{,}232{,}000$$

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Solution

c. the first letter must be a vowel (a, e, i, o, u), the first digit cannot be 0, and repetition of letters and digits is not permitted.

$$\underset{\text{L}}{5}\quad \underset{\text{L}}{25}\quad \underset{\text{L}}{24}\quad \underset{\text{D}}{9}\quad \underset{\text{D}}{9}\quad \underset{\text{D}}{8}$$

$$= 5 \cdot 25 \cdot 24 \cdot 9 \cdot 9 \cdot 8$$
$$= 1{,}944{,}000$$

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Permutations

- A **permutation** is any *ordered arrangement* of a given set of objects.

Number of Permutations

- The number of permutations of n distinct items is n factorial, symbolized $n!$, where

$$n! = n(n-1)(n-2) \cdot \cdot \cdot (3)(2)(1)$$

$$0! = 1 \text{ (This is a definition)}$$

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Example

- How many ways can 6 different stuffed animals be arranged in a line on a shelf?

$$6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$$

The 6 stuffed animals can be arranged in 720 different ways.

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Example

- Consider the six numbers 1, 2, 3, 4, 5 and 6. In how many distinct ways can three numbers be selected and arranged if repetition is not allowed?

$$6 \cdot 5 \cdot 4 = 120$$

Thus, there are 120 different possible ordered arrangements, or permutations.

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Permutation Formula

- The number of permutations possible when r objects are selected from n objects is found by the **permutation formula**

$$_nP_r = \frac{n!}{(n-r)!}$$

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Example

- The swimming coach has 8 swimmers who can compete in a “new” 100m relay (butterfly, backstroke, free style), he must select 3 swimmers, one for each leg of the relay in the event. In how many ways could he select the 3 swimmers?

$$
\begin{aligned}
{}_8P_3 &= \frac{8!}{(8-3)!} = \frac{8!}{5!} \\
&= \frac{8 \cdot 7 \cdot 6 \cdot \cancel{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}}{\cancel{5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}} \\
&= 336
\end{aligned}
$$

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Permutations of Duplicate Objects

- The number of distinct permutations of n objects where n_1 of the objects are identical, n_2 of the objects are identical, …, n_r of the objects are identical is found by the formula

$$\frac{n!}{n_1!n_2!\cdots n_r!}$$

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Example

- In how many different ways can the letters of the word “CINCINNATI” be arranged?
- Of the 10 letters, 2 are C’s, 3 are N’s, and 3 are I’s.

$$\frac{10!}{3!3!2!} = \frac{10 \times 9 \times 8 \times 7 \times 6 \times 5 \times \overset{2}{4} \times 3 \times 2 \times 1}{3 \times 2 \times 1 \times 3 \times 2 \times 1 \times 2 \times 1}$$

$$= 50,400$$

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Example Problems

- ${}_6P_4$ =
  - $\frac{6!}{(6-4)!}$ = 6 Math-> PRB -> 2 -> 4 -> Enter
  - = 360
- ${}_8P_0$ =
  - $\frac{8!}{(8-0)!}$ = 8 Math-> PRB -> 2 -> 0 -> Enter
  - = 1

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Example Problems

- Assume that a password to log onto a computer account is to consist of any four digits or letters (repetition is permitted). Determine the number of passwords possible if
  a) the letters are not case sensitive (that is, a lower case letters is treated the same as an uppercase letter).
  b) the letters are case sensitive (that is, an upper case letter is considered different than the same lower case letter).

a) 1,679,616 b) 14,776,336

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Example Problems

- In one question of a history test, a student is asked to match 10 dates with 10 events; each date can only be matched with one event. In how many ways can this question be answered?
  - (3,628,800)

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Example Problems

- In how many ways can the digits of the number 9,876,678 be arranged?
  - (630)
- A track meet has 15 participants for the 100-meter event. The 6 participants with the lowest times will be listed, in order of their times, on the leader board. How many different ways are there for the names to be listed?
  - (3,603,600)

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